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FINAL REPORT ON AFOSR GRANT 82-0305 SEQUENTIAL DECISION MODELS IN RELIABILITY

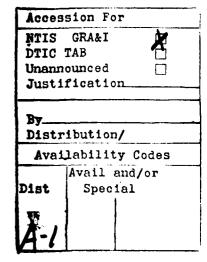
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I. INTRODUCTION AND SUMMARY

AFOSR Grant 82-0305 entitled "Sequential Decision Model in Reliability" was completed September 30, 1985. It began October 1, 1982, and was renewed twice. The research topics included Optimal Stockage Policies For Parts Which Replace Failed Components, Optimal Inspection Policies, Quality Control, and Queueing Theory Applications to Inventory. This research has helped support three articles which have either appeared in print or are accepted for publication, two technical reports and a nearly completed Ph.D. dissertation. In addition, at least one paper from this research will be submitted for publication in the next few months. Bruce Miller has been Principal Investigator throughout. Professors Eduardo Subelman, Stephen Jacobsen, and Richard Mortensen have served as Principal Investigators at various times from October 1, 1982, to September 30, 1985.

The common thread of this research is the sequential or dynamic aspect of reliability problems. This dynamic aspect showed up in various ways: What order to examine components of a failed part. What ordering rules to use for the failed parts, taking into account that it is possible to order again later in the future. The mathematical technique applied to the above problems was dynamic programming. When we shifted attention to quality control and the production of components, the dynamic aspect of the problem was still paramount. Now the question was to determine the quality level of a production process using past quality data. The mathematical technique applied to this problem was time series. We have also considered the related decision problem of when the production process should be corrected.

II. OPTIMAL STOCKAGE POLICIES

A major research effort was made on the stockage policies for spare parts which replace failed components. In research by P.I. Miller, the failure process (demands of spare parts) is described as an exponential smoothing process. Using a procedure first developed by Scarf (1960) for a Bayesian inventory model, we are able to formulate the model as a one state variable dynamic program. This is justified by Theorem 1. The other major result of this paper is Theorem 2, which says that the model we have developed will order less stock than a comparable standard formulation which assumes that demands (failures) each period are independent. This paper has been tentatively accepted by Operations Research for publication.

The paper by Faculty Associate, Katy Azoury, entitled "Bayes Solution to Dynamic Inventory Models Under Unknown Demand Distribution" was also supported by this contract. The paper of Scarf (1960) cited in the previous paragraph shows how a Bayesian inventory model can be reduced from two state variables to one in the case of the gamma demand distribution. Azoury shows how the same simplification can be obtained in the case of the uniform, Weibull, and normal demand distributions. The paper has appeared in the September, 1985, issue of Management Science.

III. QUALITY CONTROL

For the last year and a half, our major research has been to develop a dynamic statistical quality control procedure to monitor production performance. The impetus for this research comes from the Quality Measurement Plan (QMP) which was implemented in 1980 at Western Electric and is described by its originator, Hoadley. QMP is a dynamic Bayesian statistical quality control model, and it represents both a major practical and theoretical advance

over the traditional control chart approach. At the same time, QMP is recognized by its author as a first effort in a new area, and further research is continuing at Western Electric on dynamic statistical quality control. It is a measurement plan and is not explicitly a decision model. It is used as an input by management to decide which products need corrective action. The essence of the model is that in each period t, there is an underlying defective rate a_t . The number of defects in period t, X_t , is assumed to be Poisson, with parameter $e_t\theta_t$, where e_t is a scale factor. A scale factor is not difficult to include in Bayesian analysis; for example, see the 1984 in Management Science by Azoury and Miller. The value of θ_t is not known, of course. In the QMP, it is assumed to be an independent draw from a gamma prior distribution. Our approach has been to use the model of Smith, which generalizes to the Kalman filter, to model θ_t .

In Smith's approach, the observed failures Y_t , are Poisson with parameter θ_t . The parameter θ_t is assumed to have a gamma distribution with parameter a_t and b_t . We have $a_{t+1} = Y_t + ka_t$ and $b_{t+1} = 1 + kb_t$, where k is a parameter between 0 and 1 which must be estimated.

One objective of our research has been to extend Smith's results by finding properties of a random variable Z_t , such that $\theta_{t+1} = \theta_t + Z_t$. We have shown that Z_t equals θ_t/k times V_t , where V_t is a Beta random variable with parameters ka_t and $(1-k)a_t$ has the desired properties. One particular benefit of this result is that we can simulate the process of Smith where before that was not possible.

The predictive power of this approach based on Smith has been compared with that of QMP (described above), exponential smoothing, and the Kalman filter, using both quality control data and simulated stochastic processes. The final results will be described in the forthcoming Ph.D. dissertation of

Marygail Brauner. It is anticipated that some of these results will be submitted for publication. An infinite horizon of dynamic programming formulation of the quality control problem of when to correct has also been carried out.

P.I. Mortensen has always worked on quality control. He has shown that the interesting work of M.S. Phadke, which appeared in the <u>Bell Systems Technical Journal</u> is flawed in important ways. He then gives an alternate derivation. This research is described in the report entitled, "Critique of Quality Evaluation Plan Using Adaptive Kalman Filtering."

IV. OPTIMAL INSPECTION

The research in optimal inspection has been completed and the results constitute the paper of P.I. Subelman. The model is described in terms of searching for a lost object which, in reliability applications, represents a failed component. There is a known (prior) probability p_{i} , that the object is in location i. Searcher j, $1 \leq j \leq k$, is available for a total of m_{j} searches, and the cost of assigning him to location i is c_{ij} . If searcher j is assigned to location i and the lost object is in location i, the probability that the object will be found is $1 - \beta_{ij}$. In reliability applications, β_{ij} corresponds to the probability that a malfunctioning component would pass inspection. The most important results include the proposition on page 8, which gives a condition which guarantees that a box will be searched in the difficult non-identical searcher case, and the proposition and corollary on page 12. Together, they describe a situation where a myopic policy leads to minimizing the expected number of searches using non-identical searchers.

V. QUEUEING THEORY APPLICATIONS TO INVENTORY

Faculty Associate, Katy Azoury, has co-authored a paper with Professor Percy Brill entitled, "An Application of the System -Point Method to Inventory Models Under Continuous Reviews." The research has been accepted for publication in the <u>Journal of Applied Probability</u>.

This research considers continuous-review inventory models under (nQ, r) and (s, S) ordering policies in which the product is decaying. These models are formulated for a general decay function, a renewal demand process, random demand sizes and zero lead-time for ordering. A functional equation for the stationary probability density (pdf) of inventory level is derived. In this generality, the solution requires numerical methods. An analytical solution ts equation is obtained for the case where decay is a linear function of the inventory level ("exponential decay"), and demand sizes are exponentially distributed. Exponential decay is characteristic of a number of inventory applications. It is also shown that the pdf has a form which is a linear combination of a gamma and an incomplete gamma function.

Knowledge of the stationary pdf is very useful for computing various operating characteristics of the inventory system as well as the expected cost of ordering and holding inventory. The optimal policy is then determined by minimizing the cost function. For a special case of the decay function, we derived and optimized this cost function, determined the optimal stocking policy, and gave some numerical examples. This special case can also be viewed as a non-decaying inventory model with two types of random demand processes. One type is generated by very frequent demands of small sizes, while the other is much less frequent with large demand sizes.

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